Name of school: Hanson International Academy
Department: Mathematics
Course Developer: Alex X. Ning, M.T.M
Course Development Date: January 5, 2009
Course Reviser /Revision Date: Jennifer Pavey / May 20, 2010
Course Title / Grade / Course Type: Calculus and Vectors / Grade 12 / University Preparation
Ministry Course Code: MCV4U
Credit Value: 1
Curriculum Policy Document: Mathematics, Grades 11 and 12, 2010 (Revised)
Prerequisite: Advanced Functions, MHF4U
Course Description

This course builds on students’ previous experience with functions and their developing understanding of rates of change. Students will solve problems involving geometric and algebraic representations of vectors and representations of lines and planes in three dimensional space; broaden their understanding of rates of change to include the derivatives of polynomial, sinusoidal, exponential, rational, and radical functions; and apply these concepts and skills to the modeling of real-world relationships. Students will also refine their use of the mathematical processes necessary for success in senior mathematics. This course is intended for students who choose to pursue careers in fields such as science, engineering, economics, and some areas of business, including those students who will be required to take a university-level calculus, linear algebra, or physics course.

Units: Title, Descriptions and Time

<table>
<thead>
<tr>
<th>Unit</th>
<th>Titles and Descriptions</th>
<th>Time and Sequence</th>
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<tbody>
<tr>
<td>Part One</td>
<td><strong>The Geometry and Algebra of Vectors</strong></td>
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<td><strong>Unit 1</strong></td>
<td><strong>Vectors</strong>&lt;br&gt;There are four main topics pursued in this initial unit of the course. These topics are: an introduction to vectors and scalars, vector properties, vector operations and plane figure properties. Students will tell the difference between a scalar and vector quantity, they will represent vectors as directed line segments and perform the operations of addition, subtraction, and scalar multiplication on geometric vectors with and without dynamic geometry software. Students will conclude the first half of the unit by proving some properties of plane figures, using vector methods and by modeling and solving problems involving force and velocity. Next students learn to represent vectors as directed line segments and to perform the operations of addition, subtraction, and scalar multiplication on geometric vectors with and without dynamic geometry software. The final topic involves students in proving some properties of plane figures using vector methods.</td>
<td>12.5 hours</td>
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<tr>
<td>Unit 2</td>
<td><strong>Vector Applications</strong></td>
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<td>Cartesian vectors are represented in two-space and three-space as ordered pairs and triples, respectively. The addition, subtraction, and scalar multiplication of Cartesian vectors are all investigated in this unit. Applications involving work and torque are used to introduce and lend context to the dot and cross products of Cartesian vectors. The vector and scalar projections of Cartesian vectors are written in terms of the dot product. The properties of vector products are investigated and proven. These vector products will be revisited to predict characteristics of the solutions of systems of lines and planes in the intersections of lines and planes.</td>
<td>15 hours</td>
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<tr>
<th>Unit 3</th>
<th><strong>Intersection of Lines and Planes</strong></th>
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<td>This unit begins with students determining the vector, parametric and symmetric equations of lines in R² and R³. Students will go on to determine the vector, parametric, symmetric and scalar equations of planes in 3-space. The intersections of lines in 3-space and the intersections of a line and a plane in 3-space are then taught. Students will learn to determine the intersections of two or three planes by setting up and solving a system of linear equations in three unknowns. Students will interpret a system of two linear equations in two unknowns geometrically, and relate the geometrical properties to the type of solution set the system of equations possesses. Solving problems involving the intersections of lines and planes, and presenting the solutions with clarity and justification forms the next challenge. As work with matrices continues students will define the terms related to matrices while adding, subtracting, and multiplying them. Students will solve systems of linear equations involving up to three unknowns, using row reduction of matrices, with and without the aid of technology and interpreting row reduction of matrices as the creation of new linear systems equivalent to the original constitute the final two new topics of this important unit.</td>
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| Part Two | **Calculus and Rates of Change**  |
| Unit 4 | **Concepts of Calculus**  
A variety of mathematical operations with functions are needed in order to do the calculus of this course. This unit begins with students developing a better understanding of these essential concepts. Students will then deal with rates of change problems and the limit concept. While the concept of a limit involves getting close to a value but never getting to the value, often the limit of a function can be determined by substituting the value of interest for the variable in the function. Students will work with several examples of this concept. The indeterminate form of a limit involving factoring, rationalization, change of variables and one-sided limits are all included in the exercises undertaken next in this unit. To further investigate the concept of a limit, the unit briefly looks at the relationship between a secant line and a tangent line to a curve. To this point in the course students have been given a fixed point and have been asked to find the tangent slope at that value, in this section of the unit students will determine a tangent slope function similar to what they had done with a secant slope function. Sketching the graph of a derivative function is the final skill and topic. |
| 18 hours |

| Unit 5 | **Derivatives**  
The concept of a derivative is, in essence, a way of creating a short cut to determine the tangent line slope function that would normally require the concept of a limit. Once patterns are seen from the evaluation of limits, rules can be established to simplify what must be done to determine this slope function. This unit begins by examining those rules including: the power rule, the product rule, the quotient rule and the chain rule followed by a study of the derivatives of composite functions. The next section is dedicated to finding the derivative of relations that cannot be written explicitly in terms of one variable. Next students will simply apply the rules they have already developed to find higher order derivatives. As students saw earlier, if given a position function, they can find the associated velocity function by determining the derivative of the position function. They can also take the second derivative of the position function and create a rate of change of velocity function that is more commonly referred to as the acceleration function which is where this unit ends. |
| 13 hours |
| Unit 6 | **Curve Sketching**  
In previous math courses, functions were graphed by developing a table of values and smooth sketching between the values generated. This technique often hides key detail of the graph and produces a dramatically incorrect picture of the function. These missing pieces of the puzzle can be found by the techniques of calculus learned thus far in this course. The key features of a properly sketched curve are all reviewed separately before putting them all together into a full sketch of a curve. | 6.5 hours |
| Unit 7 | **Derivative Applications and Related Rates**  
A variety of types of problems exist in this unit and are generally grouped into the following categories: Pythagorean Theorem Problems (these include ladder and intersection problems), Volume Problems (these usually involve a 3-D shape being filled or emptied), Trough Problems, Shadow problems and General Rate Problems. During this unit students will look at each of these types of problems individually. | 8 hours |
| Unit 8 | **Derivative of Exponents and Log Functions-Exponential Functions**  
This unit begins with examples and exercises involving exponential and logarithmic functions using Euler’s number (e). But as students have already seen, many other bases exist for exponential and logarithmic functions. Students will now look at how they can use their established rules to find the derivatives of such functions. The next topic should be familiar as the steps involved in sketching a curve that contains an exponential or logarithmic function are identical to those taken in the curve sketching unit studied earlier in the course. Because the derivatives of some functions cannot be determined using the rules established so far in the course, students will need to use a technique called logarithmic differentiation which is introduced next. | 7 hours |
| Unit 8 | **Trig Differentiation and Application**  
A brief trigonometry review kicks off this unit. Then students turn their attention to special angles and the CAST rule which has been developed to identify which of the basic trigonometric ratios is positive and negative in the four quadrants. Students will then solve trigonometry equations using the CAST rule to locate other solutions. Two fundamental trigonometric limits are investigated for the concepts of trigonometric calculus to be fully understood. The unit ends, as in all other units in the course, with an assignment and a unit quiz. | 8 hours |
|       | **Final Evaluation** | 3 hours |
Teaching / Learning Strategies
Throughout this course students will:

- **Problem solve:** by developing, selecting, applying, and adapting a variety of problem-solving strategies;
- **Reason and prove:** by developing and applying reasoning skills to make mathematical conjectures, assess conjectures, and justify conclusions, plan and construct mathematical arguments;
- **Reflect:** by monitoring their thinking to help clarify understanding as they complete an investigation or problem;
- **Select tools and computational strategies:** by selecting and using a variety of concrete, visual, and electronic learning tools and computational strategies;
- **Connect:** by relating mathematical ideas to situations or phenomena drawn from other contexts;
- **Represent:** by making representations (e.g. Numeric, geometric, algebraic, graphical, pictorial and onscreen);
- **Communicate:** by thinking orally, visually and in writing using precise mathematical vocabulary and conventions.

Teachers will employ guided exploration, visuals, model analysis, direct instruction, problem posing and self-assessment to enable these student strategies.

Assessment and Evaluation Strategies of Student Performance

Assessment is a systematic process of collecting information or evidence about a student’s progress towards meeting the learning expectations. Assessment is embedded in the instructional activities throughout a unit. The expectations for the assessment tasks are clearly articulated and the learning activity is planned to make that demonstration possible. This process of beginning with the end in mind helps to keep focus on the expectations of the course. The purpose of assessment is to gather
the data or evidence and to provide meaningful feedback to the student about how to improve or sustain the performance in the course. Scaled criteria designed as rubrics are often used to help the student to recognize their level of achievement and to provide guidance on how to achieve the next level. Although assessment information can be gathered from a number of sources (the student himself, the student’s course mates, the teacher), evaluation is the responsibility of only the teacher. For evaluation is the process of making a judgment about the assessment information and determining the percentage grade or level.

The Final Grade
The evaluation for this course is based on the student’s achievement of curriculum expectations and the demonstrated skills required for effective learning.

The percentage grade represents the quality of the student's overall achievement of the expectations for the course and reflects the corresponding level of achievement as described in the achievement chart for the discipline.

A credit is granted and recorded for this course if the student’s grade is 50% or higher. The final grade for this course will be determined as follows:

70% of the grade will be based upon evaluations conducted throughout the course. This portion of the grade will reflect the student's most consistent level of achievement throughout the course, although special consideration will be given to more recent evidence of achievement.

30% of the grade will be based on a final exam administered at the end of the course. The exam will contain a summary of information from the course and the student’s reports and will consist of well-formulated multiple choice questions. These will be evaluated using a checklist.

The report card will focus on two distinct but related aspects of student achievement; the achievement of curriculum expectations and the development of learning skills. The report card will contain separate sections for the reporting of these two aspects.

Achievement Chart

<table>
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<tr>
<th>Categories</th>
<th>50-59% (Level 1)</th>
<th>60-69% (Level 2)</th>
<th>70-79% (Level 3)</th>
<th>80-100% (Level 4)</th>
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<tr>
<td>Knowledge and Understanding - Subject-specific content acquired in each course (knowledge), and the comprehension of its meaning and significance (understanding)</td>
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<tr>
<td>The student:</td>
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<tr>
<td>Knowledge of content (e.g., facts, terms, procedural skills, use of tools)</td>
<td>demonstrate limited knowledge of content</td>
<td>demonstrate some knowledge of content</td>
<td>demonstrate considerable knowledge of content</td>
<td>demonstrate thorough knowledge of content</td>
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<tr>
<td>Understanding of mathematical concepts</td>
<td>demonstrate limited understanding of content</td>
<td>demonstrate some understanding of content</td>
<td>demonstrate considerable understanding of content</td>
<td>demonstrate thorough and insightful understanding of content</td>
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**Thinking - The use of critical and creative thinking skills and/or processes**

| The student: | uses planning skills with limited effectiveness | uses planning skills with moderate effectiveness | uses planning skills with considerable effectiveness | uses planning skills with a high degree of effectiveness |
| Use of planning skills -understanding the problem (e.g., formulating and interpreting the problem, making conjectures) -making a plan for problem solving | uses processing skills with limited effectiveness | uses processing skills with some effectiveness | uses processing skills with considerable effectiveness | uses processing skills with a high degree of effectiveness |
| Use of processing skills -carrying out a plan (e.g., collecting data, questioning, testing, revising, modeling, solving, inferring, forming conclusions) -looking back at the solution (e.g., evaluating reasonableness, making convincing arguments, reasoning, justifying, proving, reflecting) | uses critical / creative thinking processes with limited effectiveness | uses critical / creative thinking processes with some effectiveness | uses critical / creative thinking processes with considerable effectiveness | uses critical / creative thinking processes with a high degree of effectiveness |

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### Communication - The conveying of meaning through various forms

<table>
<thead>
<tr>
<th>The student:</th>
<th>expresses and organizes mathematical thinking with limited effectiveness</th>
<th>expresses and organizes mathematical thinking with some effectiveness</th>
<th>expresses and organizes mathematical thinking with considerable effectiveness</th>
<th>expresses and organizes mathematical thinking with a high degree of effectiveness</th>
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<tr>
<td><strong>Expression and organization of ideas and mathematical thinking</strong> (e.g., clarity of expression, logical organization), using oral, visual, and written forms (e.g., pictorial, graphic, dynamic, numeric, algebraic forms; concrete materials)</td>
<td>communicates for different audiences and purposes with limited effectiveness</td>
<td>communicates for different audiences and purposes with some effectiveness</td>
<td>communicates for different audiences and purposes with considerable effectiveness</td>
<td>communicates for different audiences and purposes with a high degree of effectiveness</td>
</tr>
<tr>
<td><strong>Communication for different audiences</strong> (e.g., peers and teachers) and purposes (e.g., to present data, justify a solution, express a mathematical argument) in oral, visual, and written forms</td>
<td>uses conventions, vocabulary, and terminology of the discipline with limited effectiveness</td>
<td>uses conventions, vocabulary, and terminology of the discipline with some effectiveness</td>
<td>uses conventions, vocabulary, and terminology of the discipline with considerable effectiveness</td>
<td>uses conventions, vocabulary, and terminology of the discipline with a high degree of effectiveness</td>
</tr>
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</table>

### Application - The use of knowledge and skills to make connections within and between various contexts

| The student:                                                                 | uses conventions, vocabulary, and terminology of the discipline with limited effectiveness | uses conventions, vocabulary, and terminology of the discipline with some effectiveness | uses conventions, vocabulary, and terminology of the discipline with considerable effectiveness | uses conventions, vocabulary, and terminology of the discipline with a high degree of effectiveness |
Program Planning Considerations for Mathematics
Teachers who are planning a program in Mathematics must take into account considerations in a number of important areas. The areas of concern to all teachers that are outlined there include the following:
• types of secondary school courses
• education for exceptional students
• the role of technology in the curriculum
• English as a second language (ESL) and English literacy development (ELD)
• career education
• cooperative education and other workplace experiences
• health and safety

Considerations relating to the areas listed above that have particular relevance for program planning in Mathematics are noted here.

Education for Exceptional Students

In planning courses in Mathematics, teachers should take into account the needs of exceptional students as set out in their Individual Education Plan. All Mathematics courses reflect the real world very closely, which offers a vast array of opportunities for exceptional students. Students who use alternative techniques for communication may find a venue for their talents as they go about researching the nature of their world.

The Role of Technology in the Curriculum

Information technology is considered a learning tool that must be accessed by Mathematics students when the situation is appropriate. As a result, students will develop transferable skills through their experience with word processing, internet research, presentation software, and equation editors as would be expected in any environment.

English as a Second Language and English Literacy Development (ESL/ELD)

This Mathematics course can provide a wide range of options to address the needs of ESL/ELD students. Assessment and evaluation exercises will help ESL students in mastering the English language and all of its idiosyncrasies. In addition, since all occupations require employees with a wide range of English skills and abilities, many students will learn how the operation of their own physical world can contribute to their success in their social world.
Career Education

Mathematics definitely helps prepare students for employment in a huge number of diverse areas - Engineering, Science, Business, etc. The skills, knowledge and creativity that students acquire through this course are essential for a wide range of careers. Being able to express oneself in a clear concise manner without ambiguity, solve problems, make connections between this Mathematics course and the larger world, etc., would be an overall intention of this Mathematics course, as it helps students prepare for success in their working lives.

Cooperative Education and Other Workplace Experiences

By applying the skills they have developed, students will readily connect their classroom learning to real-life activities in the world in which they live. Cooperative education and other workplace experiences will broaden their knowledge of employment opportunities in a wide range of fields. In addition, students will increase their understanding of workplace practices and the nature of the employer-employee relationship. Teachers of Mathematics should maintain links with community-based workers to ensure that students have access to hands-on experiences that will reinforce the knowledge they have gained in school.

Health and Safety

The Mathematics program provides the reading and analytical skills for the student to be able to explore the variety of concepts relating to health and safety in the workplace. Teachers who provide support for students in workplace learning placements need to assess placements for safety and ensure that students can read and understand the importance of issues relating to health and safety in the workplace.